

What Is Claimed Is:

1 1. A method for using a computer system to solve an unconstrained
2 interval global optimization problem specified by a function f , wherein f is a scalar
3 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots x_n)$, the method comprising:
4 receiving a representation of the function f at the computer system;
5 storing the representation in a memory within the computer system; and
6 performing an interval global optimization process to compute guaranteed
7 bounds on a globally minimum value of the function $f(\mathbf{x})$;
8 wherein performing the interval global optimization process involves,
9 applying term consistency over a subbox \mathbf{X} , and
10 excluding any portion of the subbox \mathbf{X} that violates term
11 consistency.

1 2. The method of claim 1, wherein applying term consistency
2 involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a first term, $g(\mathbf{x}')$, thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$,
5 wherein the first term $g(\mathbf{x}')$ can be analytically inverted to produce an inverse
6 function $g^{-1}(\mathbf{y})$;
7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(\mathbf{X}') = h(\mathbf{X})$;
9 solving for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and
10 intersecting \mathbf{X}' with the subbox \mathbf{X} to produce a new subbox \mathbf{X}^+ ;
11 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
12 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
13 the size of the subbox \mathbf{X} .

1 3. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox for which $f(\mathbf{x}) > f_bar$;
5 applying term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} ;
6 and
7 excluding any portion of the subbox \mathbf{X} that violates the inequality.

1 4. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
5 removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away
6 from zero, thereby indicating that the subbox does not include a global minimum
7 of $f(\mathbf{x})$; and
8 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$
9 over the subbox \mathbf{X} ; and
10 excluding any portion of the subbox \mathbf{X} that violates a component.

1 5. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
4 function $f(\mathbf{x})$;
5 removing from consideration any subbox for which a diagonal element of
6 the Hessian is always negative, which indicates that the f is not convex and
7 consequently does not contain a global minimum within the subbox;

8 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
 9 subbox \mathbf{X} ; and
 10 excluding any portion of the subbox \mathbf{X} that violates an inequality.

1 6. The method of claim 1, wherein performing the interval global
 2 optimization process involves performing the Newton method, wherein
 3 performing the Newton method involves:
 4 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the function f evaluated as a function of
 5 a point \mathbf{x} over the subbox \mathbf{X} ;
 6 computing an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$; and
 7 using the approximate inverse \mathbf{B} to analytically determine the system
 8 $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes
 9 components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
 10 applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for
 11 each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} ; and
 12 excluding any portion of the subbox \mathbf{X} that violates a component.

1 7. The method of claim 1, further comprising terminating attempts to
 2 further reduce the subbox \mathbf{X} when:
 3 the width of \mathbf{X} is less than a first threshold value; and
 4 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 8. A computer-readable storage medium storing instructions that
 2 when executed by a computer cause the computer to perform a method for using a
 3 computer system to solve an unconstrained interval global optimization problem
 4 specified by a function f , wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots$
 5 $x_n)$, the method comprising:

6 receiving a representation of the function f at the computer system;
7 storing the representation in a memory within the computer system; and
8 performing an interval global optimization process to compute guaranteed
9 bounds on a globally minimum value of the function $f(\mathbf{x})$;
10 wherein performing the interval global optimization process involves,
11 applying term consistency over a subbox \mathbf{X} , and
12 excluding any portion of the subbox \mathbf{X} that violates term
13 consistency.

1 9. The computer-readable storage medium of claim 8, wherein
2 applying term consistency involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a first term, $g(\mathbf{x}')$, thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$,
5 wherein the first term $g(\mathbf{x}')$ can be analytically inverted to produce an inverse
6 function $g^{-1}(\mathbf{y})$;
7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(\mathbf{X}') = h(\mathbf{X})$;
9 solving for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and
10 intersecting \mathbf{X}' with the subbox \mathbf{X} to produce a new subbox \mathbf{X}^+ ;
11 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
12 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
13 the size of the subbox \mathbf{X} .

1 10. The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox for which $f(\mathbf{x}) > f_bar$;

1 applying term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} ;
 2 and
 3 excluding any portion of the subbox \mathbf{X} that violates the inequality.

1 11. The computer-readable storage medium of claim 8, wherein
 2 performing the interval global optimization process involves:
 3 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
 4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
 5 removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away
 6 from zero, thereby indicating that the subbox does not include a global minimum
 7 of $f(\mathbf{x})$; and
 8 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$
 9 over the subbox \mathbf{X} ; and
 10 excluding any portion of the subbox \mathbf{X} that violates a component.

1 12. The computer-readable storage medium of claim 8, wherein
 2 performing the interval global optimization process involves:
 3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
 4 function $f(\mathbf{x})$;
 5 removing from consideration any subbox for which a diagonal element of
 6 the Hessian is always negative, which indicates that the f is not convex and
 7 consequently does not contain a global minimum within the subbox;
 8 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
 9 subbox \mathbf{X} ; and
 10 excluding any portion of the subbox \mathbf{X} that violates an inequality.

13. The computer-readable storage medium of claim 8, wherein performing the interval global optimization process involves performing the Newton method, wherein performing the Newton method involves:

- computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the function f evaluated as a function of a point \mathbf{x} over the subbox \mathbf{X} ;
- computing an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$; and
- using the approximate inverse \mathbf{B} to analytically determine the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
- applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} ; and
- excluding any portion of the subbox \mathbf{X} that violates a component.

14. The computer-readable storage medium of claim 8, wherein the method further comprises terminating attempts to further reduce the subbox \mathbf{X} when:

- the width of \mathbf{X} is less than a first threshold value; and
- the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

15. An apparatus that solves an unconstrained interval global optimization problem specified by a function f , wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the apparatus comprising:

- a receiving mechanism that is configured to receive a representation of the function f ;
- a memory for storing the representation; and

7 an interval global optimization mechanism that is configured to perform
8 an interval global optimization process to compute guaranteed bounds on a
9 globally minimum value of the function $f(\mathbf{x})$;
10 a term consistency mechanism within the interval global optimization
11 mechanism that is configured to,
12 apply term consistency over a subbox \mathbf{X} , and to
13 exclude any portion of the subbox \mathbf{X} that violates term
14 consistency.

1 16. The apparatus of claim 15, wherein the term consistency
2 mechanism includes:
3 a symbolic manipulation mechanism that is configured to symbolically
4 manipulate an equation within the computer system to solve for a first term, $g(\mathbf{x}')$,
5 thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$, wherein the first term $g(\mathbf{x}')$
6 can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
7 a solving mechanism that is configured to,
8 substitute the subbox \mathbf{X} into the modified equation to
9 produce the equation $g(\mathbf{X}') = h(\mathbf{X})$, and to
10 solve for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and
11 an intersecting mechanism that is configured to intersect \mathbf{X}' with the
12 subbox \mathbf{X} to produce a new subbox \mathbf{X}^+ ;
13 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
14 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
15 the size of the subbox \mathbf{X} .

1 17. The apparatus of claim 15,
2 wherein the interval global optimization mechanism is configured to,

1 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$,
 2 and to
 3 remove from consideration any subbox for which
 4 $f(\mathbf{x}) > f_bar$;
 5 wherein the term consistency mechanism is configured to,
 6 apply term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over
 7 the subbox \mathbf{X} , and to
 8 exclude any portion of the subbox \mathbf{X} that violates the
 9 inequality.

1 18. The apparatus of claim 15,
 2 wherein the interval global optimization mechanism is configured to,
 3 determine a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$
 4 includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$), and to
 5 remove from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is
 6 bounded away from zero, thereby indicating that the subbox does
 7 not include a global minimum of $f(\mathbf{x})$; and
 8 wherein the term consistency mechanism is configured to,
 9 apply term consistency to each component $g_i(\mathbf{x})=0$
 10 ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$ over the subbox \mathbf{X} , and to
 11 exclude any portion of the subbox \mathbf{X} that violates a
 12 component.

1 19. The apparatus of claim 15,
 2 wherein the interval global optimization mechanism is configured to,
 3 determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the
 4 Hessian of the function $f(\mathbf{x})$, and to

5 remove from consideration any subbox for which a
6 diagonal element of the Hessian is always negative, which
7 indicates that the f is not convex and consequently does not contain
8 a global minimum within the subbox; and
9 wherein the term consistency mechanism is configured to,
10 apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$
11 ($i=1, \dots, n$) over the subbox \mathbf{X} , and to
12 exclude any portion of the subbox \mathbf{X} that violates an
13 inequality.

1 20. The apparatus of claim 15, further comprising a Newton
2 mechanism within the interval global optimization mechanism that is configured
3 to:
4 compute the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the function f evaluated as a function of a
5 point \mathbf{x} over the subbox \mathbf{X} ;
6 compute an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$; and to
7 using the approximate inverse \mathbf{B} to analytically determine the system
8 $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes
9 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
10 wherein the term consistency mechanism is configured to,
11 apply term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$
12 ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} , and to
13 exclude any portion of the subbox \mathbf{X} that violates term
14 consistency.

1 21. The apparatus of claim 15, further comprising a termination
2 mechanism that is configured to terminate attempts to further reduce the subbox **X**
3 when:
4 the width of **X** is less than a first threshold value; and
5 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.